SATERA Baseline: Identifying the Challenges of Space-Based Multilateration Systems

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***Abstract*—Air traffic control traditionally relies on Secondary Surveillance Radar (SSR) and procedural methods in areas without radar coverage. In recent years, ADS-B has gained importance as a surveillance technique that transmits the identity and position of aircraft. Space-based ADS-B (SB ADS-B) systems extend this capability by enabling transmission to satellites, particularly in remote and oceanic regions where ground stations are not feasible. Not only does this contribute to the reduction of separation between aircraft, but it also increases airspace operational efficiency by enabling more direct routes and greater availability of optimal altitudes, which in turn leads to lower greenhouse gas emissions. However, the integrity of ADS-B transmissions can be affected by either natural or intentional perturbations. To address this issue, the SATERA project aims to develop an integrity estimator for SB ADS-B systems using position estimates provided by multilateration (MLAT) systems. This work contributes to establishing the project's baseline and identifying specific challenges. A prediction tool has been developed to calculate the theoretical performance of an MLAT system using receiver stations aboard a constellation of Low Earth Orbit (LEO) satellites. The methodology involves adapting ground-based MLAT localization algorithms to the space environment, evaluating regularization techniques to mitigate potential ill-conditioning, and using advanced Kalman filter-based tracking algorithms combined with the Interacting Multiple Model (IMM) filter.**

***Keywords*—Space-based ADS-B, MLAT, Taylor series, Tikhonov regularization, Unscented Kalman Filter (UKF), Interacting Multiple Model (IMM).**

I. Introduction

Global air traffic is constantly increasing, and with the rapid recovery to pre-COVID-19 pandemic travel levels [1], the industry is facing serious congestion challenges. This situation affects both passengers at airports and Air Navigation Service Providers (ANSPs), who must manage the safe and efficient movement of aircraft. Standards and precise coordination are essential to prevent accidents and collisions in dense airspace.

Traditionally, providing air navigation services, such as Air Traffic Control (ATC), has mainly relied on Secondary Surveillance Radar (SSR) [2]. The latest SSR technology, Mode S, allows aircraft to send data to ground radar stations. While Mode S represents a significant improvement over Mode A/C SSR technologies, it is not always viable in remote or oceanic regions due to the high costs and infrastructure requirements.

The Automatic Dependent Surveillance-Broadcast (ADS-B) system was developed to provide surveillance where SSR is not feasible, particularly in remote and oceanic areas, and to complement SSR in regions where it is available. ADS-B enables electronic equipment onboard aircraft to automatically transmit their location (obtained from an airborne GNSS receiver), along with identity, altitude, speed, and other data, at 1090 MHz using a Mode S format known as 'Extended Squitter' (ES1090). This system provides fast and accurate data updates, significantly improving Air Traffic Controllers' (ATCOs) situational awareness and helping them effectively identify and resolve hazardous situations.

The implementation of ADS-B in modern ATC systems has significantly increased airspace capacity and facilitated more efficient Trajectory-Based Operations (TBO). However, long-haul flights over oceanic areas or remote regions still do not benefit from this technology, as current surveillance systems are ground-based, making it unfeasible to install and maintain surveillance infrastructure in these areas. As a result, aircraft in these areas must fly along fixed, relatively rigid airspace structures (also known as tracks) while maintaining greater horizontal separations [3]. This limits route optimization despite the high volume of air traffic.

This is where satellite-based air traffic surveillance, known as space-based ADS-B (SB ADS-B), becomes relevant. These systems are ideal for surveillance over oceanic and uninhabited areas [4], using Low Earth Orbit (LEO) satellite constellations to receive and relay ADS-B data to ground stations. SB ADS-B provides surveillance in regions where traditional systems lack coverage, with performance levels similar to those of ground-based ADS-B.

Companies such as Startical [5], Aireon [6], and Spire [7] have developed (or are in the process of developing) SB ADS-B systems, which offer high update rates on flight progress, improving route planning and optimizing fuel usage. This results in safer, more economical, greener, and more predictable flights.

However, both terrestrial and space-based ADS-B rely on position data obtained from an onboard GNSS receiver, such as GPS, which can be susceptible to jamming or spoofing, potentially degrading the quality of the information provided to ATCOs. In addition, ADS-B itself is vulnerable to spoofing, where either the aircraft or an external transmitter could send false positional information, or the ADS-B frequency could be jammed. Therefore, a secondary independent surveillance system is necessary.

A viable solution to address these limitations in space-based surveillance systems is multilateration (MLAT), which enables passive location of aircraft without relying on GNSS or ground-based radars. MLAT systems have proven effective in both airport surveillance and en-route areas. The combined use of ADS-B and MLAT systems, known as composite surveillance, offers an alternative to using ADS-B and SSR Mode S alone. With the development of SB ADS-B systems, the SATERA project aims to analyze the feasibility of implementing composite surveillance in the space environment, seeking to provide robust surveillance in areas where conventional systems cannot operate.

The main objective of the SATERA project is to develop and validate, through simulations and laboratory measurements, a space-based ADS-B + MLAT composite system using small satellites in low Earth orbit, complying with the requirements outlined in EUROCAE ED-142A for en-route sectors, with particular emphasis on the definition of MLAT-based integrity estimators for ADS-B data. This work aims to contribute to setting the baseline for the project and identify specific challenges posed by the space environment that SATERA will need to address.

This paper presents the outcomes of the initial steps toward defining the baseline, including modeling a satellite constellation designed to receive signals from aircraft in flight, adapting MLAT localization algorithms for the space environment, implementing regularization techniques to mitigate potential ill-conditioning issues, and employing advanced tracking algorithms based on the Kalman filter and the Interacting Multiple Model (IMM) filter.

The rest of this paper is organised as follows: Section II describes the methodology used, detailing the system design and the implemented algorithms. Section III presents and discusses the results, evaluating the performance of the various proposed techniques. Finally, Section IV summarizes the main conclusions of the work and identifies areas for future research.

II. Methodology

This section outlines the methodology for developing the multilateration (MLAT) system proposed in this work. It covers the principles of TDOA-based MLAT, the localization algorithms employed, the regularization techniques and tracking filters implemented, and the simulation process conducted.

A. Multilateration (MLAT)

An MLAT system is a cooperative and independent method for determining the position of a target. It involves a network of receiver stations and a central processing station (CPS) that runs algorithms to calculate the target's location. The system uses signals emitted by aircraft transponders to determine their position.

MLAT systems can leverage various features of the signals received from aircraft, including Time Difference of Arrival (TDOA), values derived from Time of Arrival (TOA), Round-Trip Delay (RTD), Angle of Arrival (AOA), and Frequency Difference of Arrival (FDOA). However, the initial work in the SATERA project has focused on TDOA-based solutions. Other measurements or combinations of measurements will be considered in the later stages of the project.

The principle of TDOA involves determining the unknown position of the target ($\boldsymbol{\theta} = [x, y, z]$) based on the differences in TOA between the i-th receiving station and a reference station (typically designated as station 1). Geometrically, this results in a hyperboloid, and the resulting function can be expressed mathematically as:

$$\widehat{TDOA}_{i,1} = \frac{1}{c}\|\boldsymbol{\theta} - \boldsymbol{\vartheta}_1\| - \frac{1}{c}\|\boldsymbol{\theta} - \boldsymbol{\vartheta}_i\| + n_{i,1} \quad , \qquad i = 2, ..., N_s \tag{1}$$

where the superscript $\widehat{\ }$ denotes an estimated value (used throughout the paper to indicate estimated or measured values, distinguishing them from exact values), $\boldsymbol{\vartheta}_i = [x_i, y_i, z_i]$ is the position of the i-th station, $n_{i,1}$ is a random noise term, generally assumed to have a zero-mean Gaussian distribution, c is the speed of light, and N_s is the total number of receiving stations. When at least four stations detect the aircraft's signal, its three-dimensional position can be estimated by calculating the intersection of the resulting hyperboloids.

The (estimated) TDOA measurement in (1) can also be expressed as the range difference measurement, as follows:

$$\widehat{m}_{i,1}(\boldsymbol{\theta}) = c\widehat{TDOA}_{i,1} = (r_i - r_1) + n_{i,1} \tag{2}$$

where $r_i = \|\boldsymbol{\theta} - \boldsymbol{\vartheta}_i\|$. To obtain a numerical estimate of $\boldsymbol{\theta}$, a localization algorithm processes the set of measurements in the form of (2) to construct and solve a system of equations. This system represents an inverse problem and can generally be expressed as follows:

$$\mathbf{G}\boldsymbol{\theta} = \widehat{\mathbf{m}} \tag{3}$$

where $\mathbf{G}$ is the coefficient matrix that represents the geometric behaviour of the system, $\boldsymbol{\theta}$ is the unknown position vector, and $\widehat{\mathbf{m}}$ is the vector of known TDOA measurements, including the additive noise term.

The accuracy of MLAT systems depends on several factors: measurement errors, the spatial distribution of the receiver stations, and the localization algorithm used to convert the TOA/TDOA measurement space into Cartesian coordinates (x, y, z). In particular, the geometry of the stations, described by $\mathbf{G}$, plays a crucial role.

Measurement errors can arise from various sources, leading to inaccuracies in the target's location. These errors can be modeled as part of the error term $n_{i,1}$ in equation (1) and are typically represented by a zero-mean Gaussian distribution with a specified standard deviation [8]. Once the standard deviations of each error source affecting a particular station have been identified, the total variance associated with that station can be calculated as follows:

$$\sigma^2_{TOA_i} = \sum_j \sigma^2_{source\ j,i} \tag{4}$$

where $\sigma^2_{TOA_i}$ is the variance of the i-th station and $\sigma^2_{source\ j,i}$ is the one corresponding to the j-th error source affecting the i-th station.

Besides the error due to the signal-to-noise ratio, as described in [8], this work also considers secondary contributions such as instrumental errors, synchronization errors, and errors in the measurement of the stations' positions (i.e., the satellites). Initially, a nominal standard deviation of $\sigma_{TOA_i} = 10^{-8}\ s$ with zero mean is assumed. Additionally, a standard deviation of 10 meters is added to each of the satellites' coordinates, assuming their positions can be determined with accuracy comparable to that obtained using onboard GNSS systems. In the Aireon system, for example, timing and positional accuracy are achieved through Precision Timing and Position (PTP) messages from Iridium satellites, which provide satellite position accuracy within 240 meters and timing accuracy within 200 nanoseconds [9].

To provide coverage for the aircraft, a satellite constellation has been designed from scratch to meet the minimum requirements for applying the MLAT technique, ensuring that the aircraft is visible from at least four satellites at all times for 3D positioning. This is crucial because current constellations do not always fulfill this requirement globally [10]. In this work, a constellation has been designed using MATLAB's Satellite Communications Toolbox, with characteristics similar to the Iridium constellation but featuring a greater number of satellites and orbital planes. The constellation consists of 160 satellites, distributed across 10 orbital planes at an inclination angle of 86.4° (Figure 1).

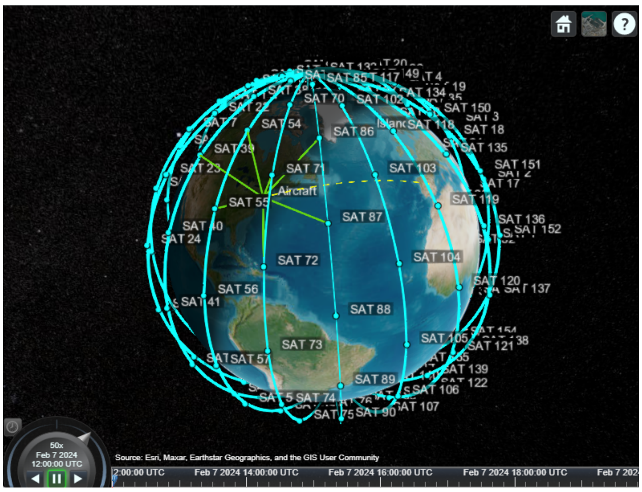


Figure 1. Satellite scenario.

B. Localization algorithms

To solve the resulting inverse problem (3), the Taylor series estimation (also known as Gauss or Gauss-Newton interpolation) [11], [12] is a widely accepted strategy for estimating the position of a target in MLAT systems. This method enables the solution of the non-linear minimization problem through an iterative approach based on Least Squares (LS). Specifically, the problem is addressed iteratively to refine the position estimate.

Let the unknown target position be denoted as $\boldsymbol{\theta} = [x, y, z]^T$. Given that the system's measurements from N_s stations are grouped into the vector $\widehat{\mathbf{m}} = [\widehat{TDOA}_{i,1}, ..., \widehat{TDOA}_{N_s,1}]^T$, the iterative formulation of the Taylor series expansion method can be expressed as:

$$\hat{\boldsymbol{\theta}}^k = \left(\mathbf{G}(\hat{\boldsymbol{\theta}}^{k-1})^T \mathbf{G}(\hat{\boldsymbol{\theta}}^{k-1})\right)^{-1} \mathbf{G}(\hat{\boldsymbol{\theta}}^{k-1})^T \widehat{\mathbf{m}}_\Delta(\hat{\boldsymbol{\theta}}^{k-1}) + \hat{\boldsymbol{\theta}}^{k-1} \qquad k = 1, ..., K \tag{5}$$

where $\hat{\boldsymbol{\theta}}^0 = \boldsymbol{\theta}_0$, $\widehat{\mathbf{m}}_\Delta(\hat{\boldsymbol{\theta}}^{k-1}) = \widehat{\mathbf{m}} - \mathbf{m}(\hat{\boldsymbol{\theta}}^{k-1})$, and K is the maximum number of Taylor iterations. This iterative process approximates the target's position $\boldsymbol{\theta}$ using a starting point or initial estimate $\boldsymbol{\theta}_0$.

The convergence of the iterative algorithm critically depends on the quality of the initial estimate. If $\boldsymbol{\theta}_0$ is not sufficiently close to the true solution, the method may not converge or may do so too slowly. Additionally, ill-conditioning problems in the matrix $\mathbf{G}$ can compromise the numerical stability of the algorithm. Therefore, having an adequate starting point is crucial.

To define such a starting point $\boldsymbol{\theta}_0$, various closed-form localization algorithms have been implemented and compared, including those proposed by Smith and Abel [13], Friedlander [14], Schau and Robinson [15], Chan and Ho [16], and Bancroft [17]. These algorithms provide initial estimates used as input for the iterative process. They differ in their approaches to linearizing the MLAT problem and vary in accuracy, robustness to measurement errors, and computational complexity. Among these, Bancroft's method has demonstrated the best results, offering a suitable initial estimate.

C. Regularization

In some scenarios, due to the system's geometry, measurement noise, or the quality of the initial estimate, the inverse problem may become ill-conditioned. This can result in the solution obtained through (5) being incorrect or diverging, leading to large errors. Numerically, this issue arises because the matrix product $(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T$, known as the pseudoinverse matrix $\mathbf{G}^\dagger$, may not satisfy the three Hadamard conditions [18] for a well-conditioned problem.

When any of these conditions are not met, the problem becomes ill-conditioned. To address these issues, it is essential to apply strategies that transform the problem into a well-conditioned one, making the solution more robust to perturbations. These strategies are commonly referred to as regularization methods.

In this work, to address the iterative procedure based on the Taylor series expansion method and to mitigate errors caused by ill-conditioning, Tikhonov regularization has been applied [19]. This technique introduces a penalty term, the regularization parameter λ, which controls the influence of perturbed data, thereby reducing the sensitivity of the final solution to slight variations or errors in the data.

The resulting regularized Taylor series solution is expressed as follows:

$$\hat{\boldsymbol{\theta}}_{\lambda}^{k} = \mathbf{A}_{\lambda}^{-1}(\hat{\boldsymbol{\theta}}_{\lambda}^{k-1})\widehat{\mathbf{m}}_{\Delta}(\hat{\boldsymbol{\theta}}_{\lambda}^{k-1}) + \hat{\boldsymbol{\theta}}_{\lambda}^{k-1}\ ,\ k = 1, ..., K \quad (6)$$

where $\mathbf{A}_{\lambda}^{-1}$ is the Tikhonov-regularized inverse matrix [20]:

$$\mathbf{A}_{\lambda}^{-1} = (\mathbf{G}^{T}\mathbf{N}(\boldsymbol{\theta})^{-1}\mathbf{G} + \lambda^{2}\mathbf{L}^{T}\mathbf{L})^{-1}\mathbf{G}^{T}\mathbf{N}(\boldsymbol{\theta})^{-1} \quad (7)$$

where $\mathbf{N}(\theta)$ is the measurement error covariance matrix, $\mathbf{L}$ is the regularization matrix, and λ is the regularization parameter. Since the covariance matrix $\mathbf{N}(\boldsymbol{\theta})$ is often unknown in practical applications because it depends on the true target position $\boldsymbol{\theta}$, it is common in practice to approximate it by assuming an identity matrix in (7).

The selection of the regularization parameter λ and the regularization matrix $\mathbf{L}$ is a critical aspect of the regularization process. The choice of the regularization matrix is closely linked to the statistical properties of the target position vector $\boldsymbol{\theta}$. When the components of $\boldsymbol{\theta}$ are assumed to be non-random and uncorrelated, a common choice for the regularization matrix is $\mathbf{L} = \mathbf{I}_{3\times3}$, where $\mathbf{I}$ denotes the identity matrix [20].

In contrast, determining the value of the regularization parameter λ is less straightforward. Numerous methods have been proposed in the literature to estimate an appropriate value for λ. In this work, the L-curve method is employed to determine the optimal value of λ [21]. This graphical technique helps in selecting the regularization parameter by identifying the optimal balance between the accuracy of the data fit and the stability of the solution.

D. *Tracking filter*

In addition to the previously mentioned methods for determining the target's position, an aircraft tracking filter has been implemented to estimate the position directly, thereby avoiding the potential issues of ill-conditioning associated with solving an inverse problem.

This tracking filter combines the Unscented Kalman Filter (UKF) with the Interacting Multiple Model (IMM). The UKF [22] utilizes the Unscented Transformation (UT) technique, which accurately captures the propagation of state and covariance without explicit linearization, thus effectively handling non-linear systems with precision and robustness. A similar approach, using the UKF alongside the Extended Kalman Filter (EKF) in a ground-based MLAT system, is described in [23].

On the other hand, the IMM [24] is an advanced filtering technique that integrates multiple dynamic models within a probabilistic framework, thereby reducing tracking errors. The primary advantage of the IMM is its ability to rapidly adapt to changes in system dynamics, enhancing accuracy without requiring precise prior knowledge of the system model. This combined approach to motion parameter estimation is often more effective than using a single UKF, especially when tracking targets that perform complex maneuvers.

Within the IMM, three models are used to accommodate different types of motion: the constant velocity (CV) model, suitable for situations where changes in the target's speed are small and random; the constant acceleration (CA) model, appropriate for representing targets with smooth movements and random acceleration changes; and the Singer model, which captures the random and changing nature of target maneuvers [25] [26].

Each model within the IMM represents a possible mode of system operation, and the filter interacts among these models to adapt to dynamic variations. Figure 2 illustrates this algorithm, which consists of the interaction of three UKF filters operating in parallel with the mentioned maneuvering models.

Since this tracking filter is a recursive filter, it is necessary to employ a method to obtain the initial step (starting the filter). To do this, the localization algorithm explained previously is used, and when three estimates of the target's position are available, the prediction of the filter can begin. With this information, the state vector is constructed, consisting of the position, velocity, and acceleration of the target at the time of initializing the tracking filter.

This approach significantly enhances tracking performance for maneuvering targets by combining the flexibility of the IMM with the non-linear estimation capabilities of the UKF. It is expected to yield improved tracking accuracy and reduced error rates, particularly during complex maneuvers. However, it is essential to note that the performance of this tracking filter may be affected by extreme environmental conditions or sensor inaccuracies, which can introduce significant challenges in the estimation process.

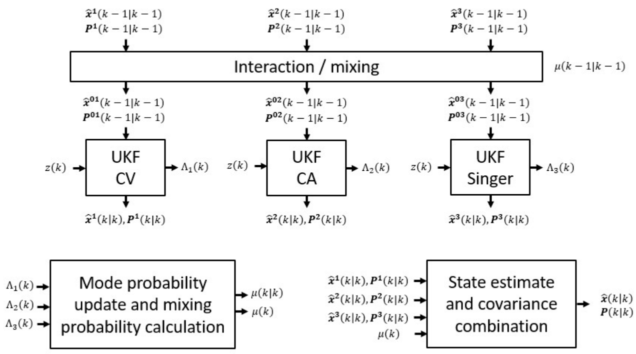


Figure 2. IMM estimator with UKF filter using CA, CV, and Singer maneuver models.

E. *Simulations*

Various validation scenarios have been conducted and evaluated for the SATERA project. The selected flight paths to assess the viability of the MLAT system include the North Atlantic (NAT) region, the Europe/South America (EUR/SAM) corridor, and the Amazon region. This article presents only the results for the North Atlantic corridor, which represents the oceanic airspace with the highest traffic density in the world (e.g., more than 1,700 operations per day over the NAT region in 2019 [27]).

The simulated route comprises a total of 100 waypoints from New York (JFK Airport) to Madrid (Adolfo Suárez Airport), Figure 3, with an ADS-B transponder transmission power of 125 W, consistent with the class A1 transmitters commonly installed in passenger-carrying aircraft to meet ATC requirements [28].

To assess system performance, the 2D RMS error and estimator bias are analyzed and discussed. Both metrics are obtained through Monte Carlo simulations with 100 trials. Additionally, the results are compared with the Cramér-Rao Lower Bound (CRLB) analysis described in [29], which establishes the theoretical minimum variance for unbiased estimators, calculated using the inverse of the Fisher Information Matrix (FIM). An efficient estimator is one that meets the CRLB, signifying that all the available information has been effectively extracted, thereby optimizing localization accuracy.

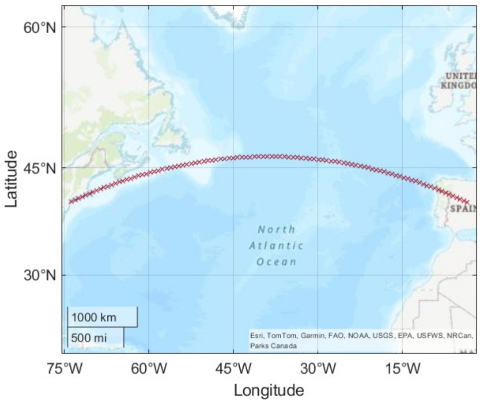
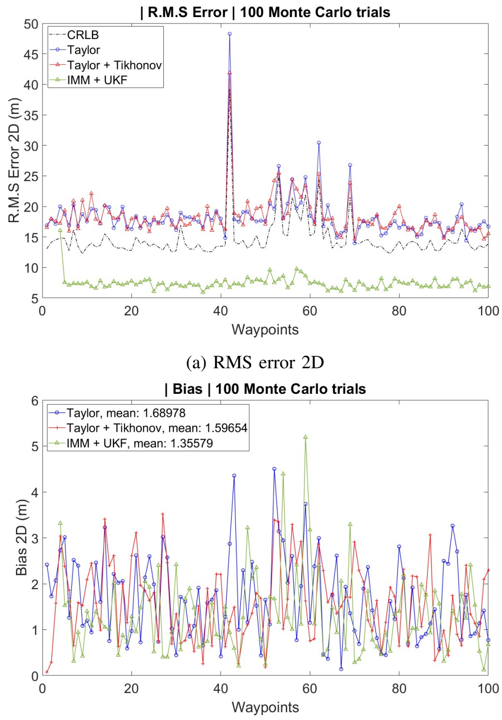


Figure 3. North Atlantic trajectory.

III. Results

This section presents the results from various simulation scenarios used to evaluate the performance of the MLAT system and the proposed localization algorithms. Several methods are compared, including the iterative Taylor series expansion approach, its regularized version using Tikhonov regularization, and the IMM+UKF tracking filter. These methods are assessed based on horizontal position error (RMS) and bias. Simulated trajectories from the NAT corridor are analyzed under different measurement error conditions, satellites position accuracy, and system geometry variations due to the satellites' movement along their orbits. The results of these simulations are discussed below.

Figure 4 displays the results for the NAT corridor, considering a nominal standard deviation of $10^{-8}s$ in the TOA measurements and a standard deviation of 10 meters in each of the satellites' coordinates, as mentioned above, with an aircraft take-off time of 12:00h. It can be observed that the Taylor method and its Tikhonov-regularized version produce similar error and bias results, both positioned slightly above the Cramer-Rao bound by a few meters. Notice that Tikhonov regularization achieves lower RMS error values at specific points along the trajectory, which suggests that some ill-conditioning may still affect the solution at those points.



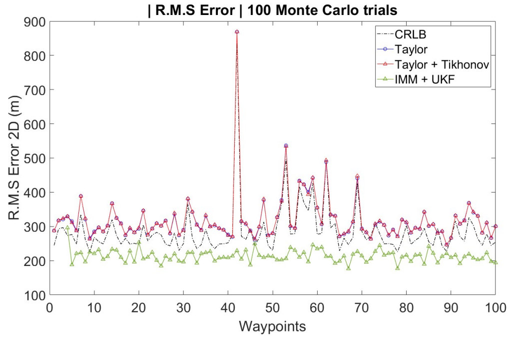
(a) RMS error 2D

(b) Bias 2D

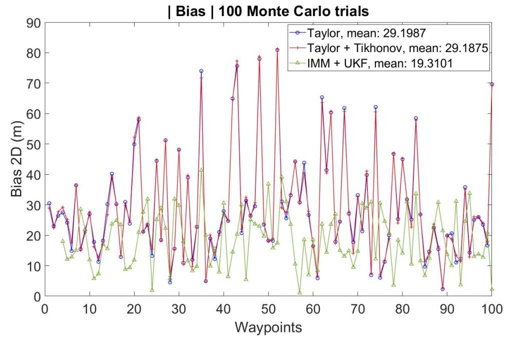
Figure 4. Comparison of the CRLB, the Taylor series method, its Tikhonov-regularized version, and the IMM+UKF filter with a $\sigma_{TOA_i} = 10^{-8}\ s$, $\sigma_{SAT} = 10\ m$ and a take-off time of 12:00h.

However, Tikhonov regularization does not entirely eliminate the error peaks, a feat achieved by the IMM+UKF filter. The IMM+UKF filter demonstrates superior performance compared to the other methods, showing the lowest bias and RMS error values at most points along the trajectory. Furthermore, this method is not affected by ill-conditioning, as it does not involve directly solving an inverse problem, but instead uses a filtering approach that adapts well to the varying geometry and measurement noise.

On the other hand, Figure 5 illustrates the results for a scenario where the measurement error at the stations was increased from $\sigma_{TOA_i} = 10^{-8}\ s$ to $\sigma_{TOA_i} = 10^{-6}\ s$, while still assuming unbiased measurements. This adjustment allows for the assessment of each method's sensitivity to random time measurement errors. It is apparent that both error and bias increased significantly across all evaluated methods, highlighting their sensitivity to noise in the time measurements.

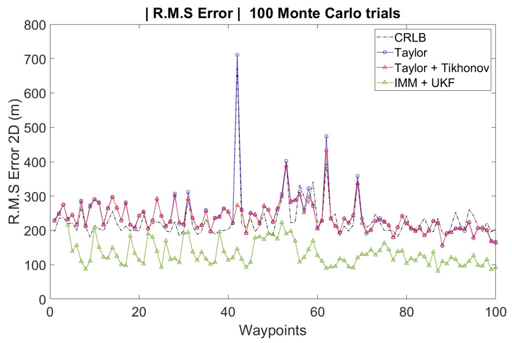


(a) RMS error 2D

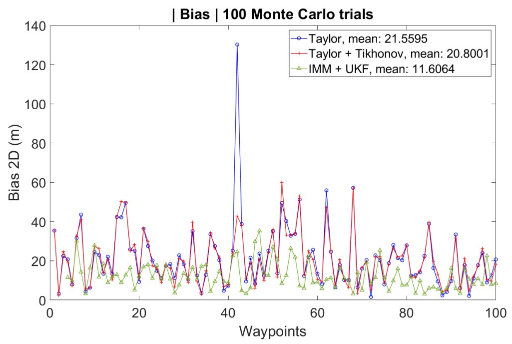


(b) Bias 2D

Figure 5. Comparison of the CRLB, the Taylor series method, its Tikhonov-regularized version, and the IMM+UKF filter with a $\sigma_{TOA_i} = 10^{-6}$ s, $\sigma_{SAT} = 10$ m and a take-off time of 12:00h.



(a) RMS error 2D



(b) Bias 2D

Figure 6. Comparison of the CRLB, the Taylor series method, its Tikhonov-regularized version, and the IMM+UKF filter with a $\sigma_{TOA_i} = 10^{-8}$ s, $\sigma_{SAT} = 240$ m and a take-off time of 12:00h.

Despite the increased noise level, the IMM+UKF filter continues to demonstrate stable performance, achieving the best error and bias values among the evaluated methods. This underscores the robustness of the IMM+UKF filter against degraded measurement quality. In contrast, the Taylor and Tikhonov methods yield similar results in terms of error and bias, suggesting that the measurement error is sufficiently high as to prevent any significant reduction in the ill-conditioning of the solution.

Additionally, Figure 6 represents the results for a scenario where the error in the satellites' coordinates is increased from 10 meters to 240 meters, similar to the case of Aireon previously mentioned [9]. This scenario allows for assessing the sensitivity of the different methods to inaccuracies in satellite positions. Consistent with the prior scenario, both error and bias increase significantly.

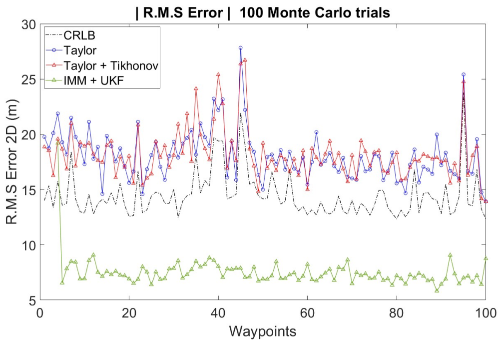
In this scenario, the IMM+UKF filter continues to exhibit superior performance, with an RMS error of approximately 150 meters and an average bias of 11.6 meters. For the Taylor and Tikhonov solutions, very similar values are observed; however, Tikhonov achieves a reduction in RMS error at certain points along the trajectory, suggesting a greater ability to handle inaccuracies in satellite positioning rather than in TOA measurements.

Moreover, it is noted that, especially towards the end of the trajectory, these two methods occasionally produce error values below the CRLB, suggesting that this limit may not be entirely reliable when errors are due to satellite position inaccuracies rather than TOA (or pseudorange) measurements. This behaviour can be explained by the fact that the error in the satellite position is added to the coefficient matrix resulting in a sort of regularization effect.

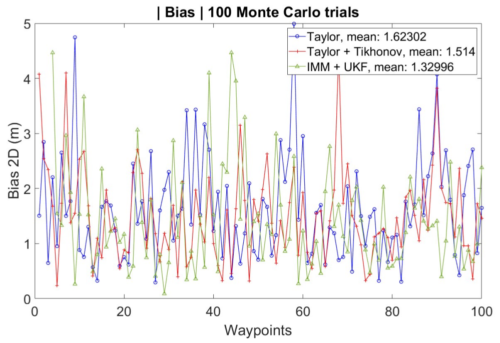
Finally, Figure 7 presents the results obtained by delaying the flight departure time by 2 hours (from 12:00h to 14:00h), allowing us to evaluate the impact of departure time changes on system performance, considering that the satellite constellation is constantly in motion and, thus, the system geometry varies dynamically.

Despite this variation in geometry, there is no significant change in the performance of the positioning methods; the results remain very similar to those from Figure 4, with nearly identical values of RMS error and bias. Peak errors occur at different points along the trajectory, attributed to changes in satellite geometry at the adjusted departure time.

In light of these results, it can be concluded that advanced tracking filters, such as the IMM+UKF, are significantly more effective in estimating position with greater accuracy and lower bias, aligning more closely with the standards set by regulations [30]. These advancements contribute to establishing the baseline for the SATERA project, laying the groundwork for enhancing the precision and accuracy of MLAT-based positioning systems in space environments. Additionally, they help identify several areas for future work and development within the project.



(a) RMS error 2D



(b) Bias 2D

Figure 7. Comparison of the CRLB, the Taylor series method, its Tikhonov-regularized version, and the IMM+UKF filter with a $\sigma_{TOA_i} = 10^{-8}$ s, $\sigma_{SAT} = 10$ m and a take-off time of 14:00h.

IV. Conclusion

This paper has evaluated various localization methods within the context of a space-based MLAT system for aircraft position estimation. The methods compared include the Taylor series expansion approach, its regularized version using Tikhonov regularization, and an advanced tracking filter that combines the Unscented Kalman Filter (UKF) with the Interacting Multiple Model (IMM). The results highlight the superior performance of advanced tracking filters, such as the IMM+UKF, especially in highly dynamic environments with noisy or ill-conditioned measurements. While Tikhonov regularization is effective in addressing ill-conditioning in certain scenarios, it does not consistently achieve the accuracy provided by the IMM+UKF filter.

Despite the promising results, factors such as satellite position errors and clock bias from synchronization could affect accuracy. Further research is needed to assess their impact. Additionally, exploring alternative localization methods that utilize measurements like Angle of Arrival (AoA) or Frequency Difference of Arrival (FDOA) could enhance space-based MLAT systems, as these methods have proven effective in ground-based systems.

This work has established a solid foundation for the SATERA project by identifying key challenges in implementing space-based MLAT positioning systems. It also highlights specific areas for further exploration, which will aid in advancing the accuracy and reliability of these systems in future developments.

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